Hide Information and Seek Liquidity: A Game of Strategic Triangular Trading

ABSTRACT

In the FX market large liquidity coexists with systemic asymmetric information. This fact is counter-intuitive. I propose a novel strategic triangular trading model where a risk-averse insider can trade currency pairs both directly and indirectly, through a third currency. In each of the three markets, the insider trader takes into account also the market liquidity in the other two markets when choosing the optimal trading strategy. Market liquidity and trading aggressiveness are sensitive to the noise trading and the private information's features. The numerical sensitivity analysis reveals limited risk-bearing capacity of the insider trader and limited cross-learning capacity of the market maker. The model could decipher the actual behaviour of better informed high-frequency trading firms and hedge funds, who have become major determinants of cross-market FX liquidity. Moreover, a solid theoretical understanding of FX market liquidity could help policy makers to better address financial stability risks affecting entire currency networks.

Keywords: FX Market Liquidity, Market Microstructure, Asymmetric Information Theory, Strategic Trading

I. Introduction

CURRENCIES ARE TRULY PUZZLING from the perspective of the asymmetric information theory. The theory is introduced in the seminal papers Kyle (1985) and Glosten and Milgrom (1985). In brief, when some market participants possess superior (private) information about the fundamental value of an asset, their trades are signals in that they reveal information to the market. In particular, the impact of trades on prices is positively linked to asymmetric information, and such impact tends to persist given the information contained in the trade. However, global foreign exchange (FX) trading is marked by small price impact, yet prominent and systemic asymmetric information. This fact is counter-intuitive.

Information asymmetries imply that a market maker runs the risk of being adversely selected (Easley, Hvidkjaer, and O'Hara (2002)) and demands an additional risk premium for trading against better informed traders (J. Wang (1993) and J. Wang (1994)). Better informed traders and insiders have always existed. For example, there is evidence of adverse selection problems in the 18th century's markets for British government annuities (Mortimer (1769)) and for cross-listed shares (Koudijs (2016)).

Although market makers cannot differentiate insiders and uninformed traders, they are aware of the adverse selection risk. In fact, they take it into account when setting market clearing prices. Intuitively, market makers compensate themselves for bad trades due to adverse selection of insiders by raising price impact or, in other words, by making the market less liquid (Bagehot (1971)). Market liquidity is a measure of how easily and quickly an asset can be bought or sold without causing large price swings (Black (1971)).

Market liquidity plays a crucial role in the functioning and smooth operation of financial markets, as it affects their efficiency and stability. High liquidity allows traders to enter and exit positions with ease, facilitating price discovery and enabling markets to absorb larger trading volumes without disrupting the overall market conditions. Low liquidity, on the other hand, can make it harder to execute trades and can create price volatility, increasing risk exposures. Vayanos and J. Wang (2011) reviewed theories of market liquidity under a unified framework to elucidate how illiquidity relates to underlying market imperfections, including asymmetric information.

The market for currencies is the world's most liquid by a large margin[1], which would suggest little information asymmetry. However, the FX market is predominantly OTC, coordinated through dealers acting as market makers and negotiating directly with brokers and customers (Liu and Y. Wang (2016)). As such, it is fragmented and opaque, with no single centralized exchange. An accurate, comprehensive and timely view of global FX transaction flows is therefore difficult to obtain. This implies that asymmetric information should be ubiquitous in FX trading, as information dispersion is inherent to FX market's unique infrastructural features[2]. Ranaldo and Somogyi (2021) provide the first direct empirical evidence of systematic asymmetric information in the FX market.

In summary, the FX market is characterised by the coexistence of large liquidity (or small price impact) and systemic asymmetric information. This fact is counter-intuitive and urges a theoretical underpinning. My broader research goal is to investigate trading on information in networks of currency pairs. At a microfoundational level, networks of currency pairs can be though of as ensembles of joint FX rates' triangles. Thus, the first step is to study trading on information in triangles of joint FX rates. To do this, I model the strategic interaction between an insider trader and a market maker — à la Kyle (1985) — in a triangular FX trading setting.

Model preview. The theoretical model characterizes trading on private information in a strategic triangular FX trading setting with insider traders, market makers, and noise traders. Such simple setting already captures the uniqueness of trading currency pairs, which are interconnected with each other through their rates. No other financial market exhibits this natural interconnection. Imagine a triangle graph where each of the three vertices represents a currency, while each of the three edges is the market for the corresponding FX rate/currency pair. As in real financial markets, FX traders can

[1] Turnover has reached \$7.5 trillion per day. See Triennal central bank survey (summary) - OTC foreign exchange turnover in April 2022, Bank for International Settlements (BIS), October 2022.

[2] These features are comprehensively described in Chaboud, Rime, and Sushko (2023).

bet on a currency pair either *directly* or *indirectly*. Betting indirectly means trading the currency pair by transacting through a third currency[3].

In the model, an insider trader receives a noisy private signal about the true log-exchange rates for two currency pairs. She can trade three currency pairs both directly and indirectly. In a strategic triangular trading game against market makers, the rational insider trader is expected to choose in a strategic way how much to trade directly versus indirectly. For both trading modes, her choices of trading aggressiveness and the associated price impacts[4] should be sensitive to the amount of noise trading in the markets and the information features of the private signal she receives. The research question answered by the model is straightforward: *how sensitive, across the three markets, are the insider's trading aggressiveness and price impacts to (i) the amount of noise trading and (ii) the signal's imprecision, asymmetry, and correlation?*

Result preview. Two underlying mechanisms drive trading on information in triangles of joint FX rates. These are the two strategic channels which regulate the interactions between the insider trader and the market makers.

The first channel is the *limited risk-bearing capacity of the insider trader*. The insider trader has mean-variance preferences, with the variance being convex. This means that the insider is willing to take a certain amount of risk but not more than that. She can distribute such risk asymmetrically by trading either directly or indirectly. Then, the more aggressive is the direct trade, the less attractive the triangle trade becomes. Without mean-variance preferences the relative attractiveness would be unaffected.

The second channel is the *limited cross-learning capacity of the market maker*. The signal has two components, A and B, each carrying private information about a specific exchange rate. Assuming zero correlation between the two components, trading A over the triangle is strategic for the insider because it makes it more difficult for the market maker who wants to learn about the other component, i.e. B. From the perspective of

[3]The "direct trade" consists in buying or selling a currency pair in the same market, while the "indirect trade" (or, interchangeably, the "triangle trade") is executed by trading such pair through its triangle of joint FX rates.

[4]For each of the three markets, the price impact represents the price adjustment by the market makers who observe the order flow in that market.

such market maker, triangle trading on A is like noise trading. However, as A becomes less valuable, the triangle trade on A becomes less likely. In turn, the same market maker would expect direct trading on B to become more likely, driving up price impact and dampening market liquidity in her market.

Policy. The characterization of the two strategic interaction channels is a preliminary but important step for better understanding key market quality attributes of interconnected FX markets and related policy issues. For example, market liquidity (or illiquidity) is a crucial market quality metric. A solid theoretical understanding of FX market liquidity helps policy makers to better address financial stability risks affecting entire networks of currency pairs[5].

The proposed model already provides a minimal explanation, based on asymmetric information, of the liquidity for three currency markets which are joint through their exchange rates. Future research on cross-market liquidity could contribute to better designs for the coordinated circuit-breakers that serve to halt illiquidity spillovers from one market to the others. These phenomena have macroeconomic relevance and implications, despite the fact that they emerge from the microstructure of financial markets[6].

Related literature. In contrast with the traditional asset pricing literature, market microstructure recognises two fundamental aspects about FX trading: some information relevant to FX rates is not publicly available, and the heterogeneity of trading agents and in trading mechanisms matters (Lyons (2001)). The simple framework I propose captures both aspects. First, private signals motivate informed trading, as I explicitly model in the strategic triangular trading setup. Second, the model includes noise traders, market makers and an insider trader.

In particular, the insider trader is allowed to transact both directly and indirectly, and the two trading mechanisms have distinct implications on the currency markets.

[5]Some examples of relevant liquidity-related issues are the inefficiencies in liquidity provision for emerging markets, commonality in liquidity, liquidity spirals, and liquidity contagion (also known as liquidity spillover).

[6]For example, currency crises are associated with sudden and dramatic drains in liquidity. These negatively affect international investors' positions and force the implementation of unconventional monetary policy and FX interventions.

This is the salient and most consequential feature of the novel model, and thus the primary contribution of this paper. There are four additional contributions to the literature on asymmetric information in FX markets.

First, the model I propose uncovers two channels that elucidate the strategic interaction between insider traders and market makers in a trading environment with noise traders. These two channels are the limited risk-bearing capacity of the insider trader and the limited cross-learning capacity of the market maker. Published theoretical models do not capture these information-based drivers and the way they are reflected in the agents' behaviours[7].

Second, for each of the three markets the insider trader takes into account also the price impacts in the other two markets when selecting her optimal trading strategy. In other words, the model demonstrates that asymmetric information hidden in the order flow has a first-order effect on market liquidity across markets. Hence, the model can guide the design of a market depth's measure based on FX order flow. Such measure would be preferable over those based on FX volume, like the estimators proposed in Hasbrouck and Levich (2019) and Ranaldo and Santucci de Magistris (2022)[8].

Third, the triangle of joint FX rates is the natural building block of a network of currencies, and the triangular model can be useful to zoom in on network-related phenomena. Hasbrouck and Levich (2021) provide evidence of a centrality premium in FX trading networks. Central dealers tend to learn more, trade more at lower costs, end earn higher expected profit. The results are consistent with the network model developed in Babus and Kondor (2018). Consider different ensembles of currencies, where each ensemble includes multiple triangles sharing a single and distinctive vertex/currency. Although the concept of centrality is technically meaningless in a triangle graph, a currency's centrality can be examined, in a stylised form, by comparing such ensembles.

[7]However, information transmission in FX markets is intuitive (Evans and Rime (2019)). End-users flows convey the information to market making dealers, who use it to manage the risk of supplying liquidity through their trades. In aggregate, these trades incorporate the information into FX rates.

[8]In particular, whenever the order flow imbalance is volatile and tends to revert, FX volume-based measures fail to capture market liquidity and price impact estimates should be adjusted to the imbalance of the order flow. Appendix A includes a broader discussion on FX market liquidity.

Fourth, the parsimony of the setup makes it flexible enough to accommodate microfoundations that explain different nuances of the empirical evidence on asymmetric information in FX markets[9]. Ranaldo and Somogyi (2021) find that asymmetric information is unevenly spread across agents, time, and currency pairs (and its risk is priced in the FX market)[10]. Cespa et al. (2022) show that trading in spot and forward markets is more informed than in swap market. The triangular trading model can easily be augmented to allow for multiple and differently informed insider traders, or for a single insider to transact, in a strategic way, in the spot, forward, or swap markets.

Layout. The paper proceeds as follows. Section II presents the strategic triangular trading model. Section III analyses the main findings and the two strategic interaction channels. Section IV concludes. Appendix A discusses FX market liquidity. Appendix B introduces the random variables' distributions. Appendices C, D, and E include proofs.

[9]Empirical studies include Perraudin and Vitale (1996), Evans (2002), Evans and Lyons (2006), Chaboud, Chernenko, and Wright (2007), Mancini, Ranaldo, and Wrampelmeyer (2013), and Menkhoff et al. (2016).

[10]The paper marks the inaugural examination of data tracking the global intraday FX order flow by category of market participant and side of trade taken. For more information about the dataset, see FX Flow Data by CLS Group, the world's largest currency settlements provider.

II. Model

This section details the theoretical setup and it includes two subsections. First, I introduce the different agents, assumptions, and the conjectured equilibrium for the model. Second, I discuss and solve the problem of the insider trader and the problem of the market maker.

A. Model's agents, assumptions and conjectured equilibrium

The model has three objectives. First, it must capture that currency traders can bet on a currency pair both in the direct market and indirectly, through the markets for a third currency. This is a peculiarity of FX trading. Second, given the two trading modes, the model must quantify the sensitivity of the insider's trading aggressiveness and price impacts to the amount of noise trading and to the signal's imprecision, asymmetry, and correlation. Third, such sensitivity must be explained by the strategic interaction channels between the insider trader and the market maker. Thus, the model must precisely identify and isolate such strategic channels.

The theoretical model for the triangular FX trading setting is an adaptation of the influential and pioneering strategic behaviour model developed in Kyle (1985). The model's essence is a hide and seek game between an insider trader and a market maker. The market maker cannot distinguish the informative orders, submitted by the insider trader, from the uninformative orders, submitted by noise traders. As a consequence, the market maker can only observe the pooled order flow and tries to seek the information from it (as the flow also embeds the informative component). However, the insider trader tries to strategically hide his information-driven trades by exploiting the uninformative flow variance, thus manipulating the degree to which the price moves against her.

Agents. The strategic triangular trading setting models the behaviours of a risk-averse informed currency trader (insider), a continuum of noise traders, and a continuum of risk-neutral market makers. In the first period, the insider receives a noisy private signal $\widetilde{\zeta} = \widetilde{\nu} + \widetilde{\epsilon}$ about two log-exchange rates $\widetilde{\nu}$ $(\perp \widetilde{\epsilon})$ and submits market orders $\widetilde{\chi}(\widetilde{\zeta})$ in three markets for currency pairs. At the same time, in the same markets, the noise traders submit a price-inelastic demand $\widetilde{u}$. In the second period, the market makers observe the pooled order flow $\widetilde{y} = \widetilde{\chi} + \widetilde{u}$ only in their respective market and quote it competitively. That is, for each of the three markets, the quotes set by the market makers are simply their expectation about the true log-exchange rate given the pooled order flow they observe: $q = \mathrm{E}[\widetilde{v}|\widetilde{y} = y] = q(y)$. All the random variables are normally distributed, which makes the model tractable. Appendix B introduces the normal distributions of the random variables.

Assumptions. The model is a static/one-shot and order-driven trading system. The placement of (market) orders happens before the quotes are set. Moreover, trading is strategic. The information monopolist explicitly takes into account the price impact of her orders when choosing the optimal quantities in the three markets. It follows that market depth and trading aggressiveness are endogenous and interacting in this model. On the contrary, noise trading is independent from the true log-exchange rates ($\widetilde{u} \perp \widetilde{v}$), as it is exogenous, uninformed, and non-speculative. If it was not, the only possible equilibrium would be fully revealing.

Market making has three important features. First, it is separate across markets. The quote in one market is based on the pooled order flow only observed in that same market. The pooled order flows in the other two markets are only observable to their respective market makers. Second, market making is characterized by batch-clearing, so the market makers cannot distinguish single trades and no bid-ask spreads exist. As the noise flow camouflages the informed flow, batch-clearing limits the learning ability of the market makers and this is strategically exploited by the insider trader to make profits. Third, unlike in rational expectations equilibrium models, market making here incarnates the auctioneers. Market makers play the explicit roles of pricing and absorbing order flow.

Equilibrium. The conjectured equilibrium in this model is defined by the trading positions/strategies χ chosen by the insider and the quotes q chosen by the market makers such that three conditions are satisfied. First, the insider optimizes mean-variance profits, given the market makers' quoting rules. Second, the market makers set the quotes to earn zero profits, given the insider's trading strategies. Third, the insider and the market makers have rational expectations[11]. At the equilibrium, the trading strategies and the quoting rules are conjectured to be the following:

$$
\begin{aligned}
\chi_{\frac{¥}{\$}} &= \beta_{11}^{+}\left(\zeta_{\frac{¥}{\$}} - \overline{v}_{\frac{¥}{\$}}\right) + \beta_{12}^{-}\left(\zeta_{\frac{\$}{€}} - \overline{v}_{\frac{\$}{€}}\right) & \ln q_{\frac{¥}{\$}} &= \overline{v}_{\frac{¥}{\$}} + \lambda_{\frac{¥}{\$}}\left(\chi_{\frac{¥}{\$}} + u_{\frac{¥}{\$}}\right) \\
\chi_{\frac{\$}{€}} &= \beta_{21}^{-}\left(\zeta_{\frac{¥}{\$}} - \overline{v}_{\frac{¥}{\$}}\right) + \beta_{22}^{+}\left(\zeta_{\frac{\$}{€}} - \overline{v}_{\frac{\$}{€}}\right) & \ln q_{\frac{\$}{€}} &= \overline{v}_{\frac{\$}{€}} + \lambda_{\frac{\$}{€}}\left(\chi_{\frac{\$}{€}} + u_{\frac{\$}{€}}\right) \\
\chi_{\frac{¥}{€}} &= \beta_{31}^{+}\left(\zeta_{\frac{¥}{\$}} - \overline{v}_{\frac{¥}{\$}}\right) + \beta_{32}^{+}\left(\zeta_{\frac{\$}{€}} - \overline{v}_{\frac{\$}{€}}\right) & \ln q_{\frac{¥}{€}} &= \overline{v}_{\frac{¥}{\$}} + \overline{v}_{\frac{\$}{€}} + \lambda_{\frac{¥}{€}}\left(\chi_{\frac{¥}{€}} + u_{\frac{¥}{€}}\right)
\end{aligned}
\tag{1}
$$

In these equations, $\chi_{\frac{¥}{\$}}$ is the USD trade quantity, $\chi_{\frac{\$}{€}}$ is the USD volume invested in EUR, and $\chi_{\frac{¥}{€}}$ is the USD value of the EUR trade in the JPYEUR market. Then, $\zeta_{\frac{¥}{\$}} - \overline{v}_{\frac{¥}{\$}}$ and $\zeta_{\frac{\$}{€}} - \overline{v}_{\frac{\$}{€}}$ are the deviations of the noisy private signals about the true values of the future log-exchange rates JPYUSD and USDEUR from their respective unconditional expected values. Finally, the log-quotes $\ln q_{\frac{¥}{\$}}$, $\ln q_{\frac{\$}{€}}$, and $\ln q_{\frac{¥}{€}}$ are the current log-exchange rates, while $u_{\frac{¥}{\$}}$, $u_{\frac{\$}{€}}$ and $u_{\frac{¥}{€}}$ represent the noise trading quantities in the three markets for the currency pairs.

The key equilibrium parameters are the βs and the λs. These capture, respectively, the trading aggressiveness and the price impact in the three markets. The nine parameters characterize the linearity of the conjectures, a key feature in this theoretical setting. Moreover, unlike in rational expectations equilibrium models, the insider trader cannot condition on the realized noise trading quantities when deciding her optimal strategies.

The three equations for the equilibrium trading strategies can be interpreted with the following convention. The insider trader receives two private signals, $\zeta_{\frac{¥}{\$}}$ and $\zeta_{\frac{\$}{€}}$, and trades on them directly and indirectly. If $\zeta_{\frac{¥}{\$}}$ shows that USD appreciates against JPY, the insider can either (i) directly buy USD against JPY (trade $\chi_{\frac{¥}{\$}}$ through β_{11}^{+}) or (ii) buy EUR against JPY (trade $\chi_{\frac{¥}{€}}$ through β_{31}^{+}) and sell EUR against USD (trade $\chi_{\frac{\$}{€}}$ through β_{21}^{-}). Similarly, if $\zeta_{\frac{\$}{€}}$ shows that EUR appreciates against USD, the insider can either

[11]The insider trader's actual equilibrium behaviour is that expected by the market makers and vice versa.

(i) directly buy EUR against USD (trade $\chi_{\frac{\$}{€}}$ through β^{+}_{22}) or (ii) buy EUR against JPY (trade $\chi_{\frac{¥}{€}}$ through β^{+}_{32}) and sell USD against JPY (trade $\chi_{\frac{¥}{\$}}$ through β^{-}_{12}).

The model's equilibrium is characterized in two stages, by proving the self-consistency of the conjectures. First, I solve the insider trader's problem for the trading aggressiveness parameters, the βs. Then, I plug the βs into the market maker's problem and solve it for the price impact parameters, the λs.

B. Insider trader problem and market maker problem

The insider trader is active in all markets and mean-variance optimizes the approximated profits denominated in USD and combined over the three markets. Since exchange rates are log-normally distributed, the profits' approximation by log-linearization is convenient in order to keep working with normally distributed log-exchange rates[12].

After the due conversions into USD, the profits for each market are defined as the corresponding trading position times the deviation of the future log-exchange rate from the current log-exchange rate. Since the insider trader is strategic, she incorporates the conjectured quoting rules of the market makers when computing profits, so the current log-exchange rates are simply the log-quotes. The USD-denominated approximated combined profits, Π^*, is a linear combination of the log-exchange rates $v_{\frac{¥}{\$}}$ and $v_{\frac{\$}{€}}$, and of the noise trading quantities $u_{\frac{¥}{\$}}$, $u_{\frac{\$}{€}}$, and $u_{\frac{¥}{€}}$. Appendix C reports the steps leading to the approximated combined profits denominated in USD.

Insider trader problem. The mean-variance optimization problem is the following:

$$\max_{\chi_{\frac{¥}{\$}},\chi_{\frac{\$}{€}},\chi_{\frac{¥}{€}}} \left\{ \mathrm{E}\left[\Pi^*|\zeta\right] - \frac{\gamma}{2}\mathrm{Var}\left[\Pi^*|\zeta\right] \right\} \tag{2}$$

The terms $\mathrm{E}\left[\Pi^*|\zeta\right]$ and $\mathrm{Var}\left[\Pi^*|\zeta\right]$ depend, respectively, on the conditional mean $\mathrm{E}\left[v|\zeta\right]$ and on the conditional variance $\mathrm{Var}\left[v|\zeta\right]$. These conditional terms follow from the application of the linear projection theorem. In particular, the conditional mean is linear in

[12]If q denotes the current exchange rate and f the future exchange rate, then the log-linear approximation $f/q - 1 \approx \ln(f/q)$ for $f \approx q$ holds.

the signals' deviations from their unconditional means through coefficients that are combinations of the (exogenously given) normal distribution parameters for the signals. The conditional variance is a 2x2 matrix of coefficients also dependent on the same signals' distribution parameters. Appendix D characterizes the insider trader problem.

All in all, since the log-quotes are conjectured to be linear in the traded quantities, the objective function is quadratic. Then, solving the system of three first order conditions verifies the linear conjecture for the three optimal trading strategies (1).

In conclusion, the trading aggressiveness (i.e., the βs) depends on the risk aversion γ, the noise traders' covariance matrix, the terms in $\mathrm{E}[\nu|\zeta]$ and $\mathrm{Var}[\nu|\zeta]$, and, most importantly, on the price impacts $\lambda_{\frac{¥}{\$}}$, $\lambda_{\frac{\$}{€}}$ and $\lambda_{\frac{¥}{€}}$. Therefore, for each market, the trader takes into account also the *cross-market impact* when selecting her optimal trading strategy.

Market maker problem. The market maker's problem consists in deriving the price impact coefficients given the trading aggressiveness found as solution to the insider trader's problem. The simplifying assumption is that market makers are separate across markets, that is, they can observe the pooled order flow only for their respective market.

For each one of the three markets, the quoting rule given perfect competition is the market makers' expectation of the true value of the future log-exchange rate conditioned on the pooled order flow they observe in that same market.

The random variables for the future log-exchange rate and for the pooled order flow are assumed to be jointly normal. Thus, the conditional expectations are simply found by applying the linear projection theorem, which implies that the best predictor is, in fact, the linear predictor.

To find the price impacts, recall that (i) noise trading is independent from the true log-exchange rates and the insider's optimal trading strategies and (ii) the expected order flow imbalance is zero. Then, the price impacts are found by matching the three liner projections with the conjectured log-quotes, plugging in the exogenous normal distributions' parameters and the βs from the insider trader problem:

$$\begin{aligned}
\lambda_{\frac{¥}{\$}} &= \frac{\beta_{11}\mathrm{Var}[v_{\frac{¥}{\$}}] + \beta_{12}\mathrm{Cov}[v_{\frac{¥}{\$}}, v_{\frac{\$}{€}}]}{\beta_{11}^2\mathrm{Var}[\zeta_{\frac{¥}{\$}}] + \beta_{12}^2\mathrm{Var}[\zeta_{\frac{\$}{€}}] + 2\beta_{11}\beta_{12}\mathrm{Cov}[\zeta_{\frac{¥}{\$}}, \zeta_{\frac{\$}{€}}] + \mathrm{Var}[u_{\frac{¥}{\$}}]} \\
\lambda_{\frac{\$}{€}} &= \frac{\beta_{21}\mathrm{Cov}[v_{\frac{¥}{\$}}, v_{\frac{\$}{€}}] + \beta_{22}\mathrm{Var}[v_{\frac{\$}{€}}]}{\beta_{21}^2\mathrm{Var}[\zeta_{\frac{¥}{\$}}] + \beta_{22}^2\mathrm{Var}[\zeta_{\frac{\$}{€}}] + 2\beta_{21}\beta_{22}\mathrm{Cov}[\zeta_{\frac{¥}{\$}}, \zeta_{\frac{\$}{€}}] + \mathrm{Var}[u_{\frac{\$}{€}}]} \\
\lambda_{\frac{¥}{€}} &= \frac{\beta_{31}\mathrm{Var}[v_{\frac{¥}{\$}}] + (\beta_{31} + \beta_{32})\mathrm{Cov}[v_{\frac{¥}{\$}}, v_{\frac{\$}{€}}] + \beta_{32}\mathrm{Var}[v_{\frac{\$}{€}}]}{\beta_{31}^2\mathrm{Var}[\zeta_{\frac{¥}{\$}}] + \beta_{32}^2\mathrm{Var}[\zeta_{\frac{\$}{€}}] + 2\beta_{31}\beta_{32}\mathrm{Cov}[\zeta_{\frac{¥}{\$}}, \zeta_{\frac{\$}{€}}] + \mathrm{Var}[u_{\frac{¥}{€}}]}
\end{aligned} \tag{3}$$

The λs are the price impacts and their reciprocals capture the market depths or, equivalently, the *market liquidity* of the three markets. The price impact equations depend on nine exogenously given parameters: γ (insider trader's risk aversion); $\sigma_{\epsilon_{\frac{¥}{\$}}}$ (information imprecision about JPYUSD rate); $\sigma_{\epsilon_{\frac{\$}{€}}}$ (information imprecision about USDEUR rate); $\sigma_{u_{\frac{¥}{\$}}}$ (noise trading in JPYUSD market); $\sigma_{u_{\frac{\$}{€}}}$ (noise trading in USDEUR market); $\sigma_{u_{\frac{¥}{€}}}$ (noise trading in JPYEUR market); $\sigma_{v_{\frac{¥}{\$}}}$ (information asymmetry about JPYUSD rate); $\sigma_{v_{\frac{\$}{€}}}$ (information asymmetry about USDEUR rate); $\rho_{u_{\frac{¥}{\$},\frac{\$}{€}}}$ (noise trading correlation in JPYUSD and USDEUR markets); $\rho_{u_{\frac{¥}{\$},\frac{¥}{€}}}$ (noise trading correlation in JPYUSD and JPYEUR markets); $\rho_{u_{\frac{\$}{€},\frac{¥}{€}}}$ (noise trading correlation in USDEUR and JPYEUR markets); $\rho_{v_{\frac{¥}{\$},\frac{\$}{€}}}$ (information correlation about JPYUSD and USDEUR rates). Appendix E characterizes the market maker problem.

In conclusion, the equilibrium is fully characterized by solving for the λs. However, these price impact equations are extremely convoluted expressions of the exogenous parameters and not yet solvable in closed-form. Therefore, the findings discussed in Section III are obtained through a numerical sensitivity analysis. For each calibration, I fix all parameters but one and analyze the sensitivity of the λs and the βs to the only parameter which I increase (progressively, in a linear manner).

III. Analysis

This section introduces a subset of the results and implications of the strategic triangular trading model. It includes three subsections and refers to two figures. In the first subsection, I discuss the sensitivities of trading aggressiveness and price impact to the progressive increase in the amount of noise trading in the JPYUSD market. In the second subsection, I reassess the sensitivities to the progressive increase in the imprecision of the signal on the JPYUSD rate. In the last subsection, I interpret these findings through the lenses of two strategic interaction channels which emerge from the analysis.

In the interest of clarity, I rewrite here the insider trader's three optimal trading strategies in an approximate and colorful form:

$$\chi_{\frac{¥}{\$}} \sim \beta_{11}^{+}\zeta_{\frac{¥}{\$}} + \beta_{12}^{-}\zeta_{\frac{\$}{€}} \quad \chi_{\frac{\$}{€}} \sim \beta_{21}^{-}\zeta_{\frac{¥}{\$}} + \beta_{22}^{+}\zeta_{\frac{\$}{€}} \quad \chi_{\frac{¥}{€}} \sim \beta_{31}^{+}\zeta_{\frac{¥}{\$}} + \beta_{32}^{+}\zeta_{\frac{\$}{€}}$$

It is also useful to rewrite their interpretation, as it would facilitate the reading of the figures even further. The insider trader receives two private signals, $\zeta_{\frac{¥}{\$}}$ and $\zeta_{\frac{\$}{€}}$, and trades on them directly and indirectly.

If $\zeta_{\frac{¥}{\$}}$ shows that USD appreciates against JPY, the insider can either (i) directly buy USD against JPY (trade $\chi_{\frac{¥}{\$}}$ through β_{11}^{+}) or (ii) buy EUR against JPY (trade $\chi_{\frac{¥}{€}}$ through β_{31}^{+}) and sell EUR against USD (trade $\chi_{\frac{\$}{€}}$ through β_{21}^{-}). Similarly, if $\zeta_{\frac{\$}{€}}$ shows that EUR appreciates against USD, the insider can either (i) directly buy EUR against USD (trade $\chi_{\frac{\$}{€}}$ through β_{22}^{+}) or (ii) buy EUR against JPY (trade $\chi_{\frac{¥}{€}}$ through β_{32}^{+}) and sell USD against JPY (trade $\chi_{\frac{¥}{\$}}$ through β_{12}^{-}).

When referring to the figures, the subscripts "1", "2", and "3" denote the three markets: JPYUSD or $\frac{¥}{\$}$, USDEUR or $\frac{\$}{€}$, and JPYEUR or $\frac{¥}{€}$. Moreover, arrows are used to better visualize different trends: increasing or $\nearrow$, decreasing or $\searrow$, and vanishing or $\to 0$.

The discussions are focused on explaining the sensitivities of trading aggressiveness and price impact, with a special emphasis on the differences between the direct trade and the indirect/triangle trade (i.e., the trade through the triangle of quotes).

A. Increased noise trading in JPYUSD

The amount of order flow necessary to raise the price by \$1 is $1/\lambda$, i.e. the market liquidity or, more specifically, the "depth" of the market. As in Kyle (1985), the higher is the proportion of noise trading to the value of the private information, the smaller the price impact or the deeper/more liquid is the market. Intuitively, the more noise trading relative to the value of insider information, the less a market maker needs to adjust the price in response to the pooled order flow. From the perspective of a market maker, adverse selection risk is lower when the order flow, due to large amounts of noise trading, is more likely to be uninformative. The following discussion is focused on the effects of increased noise trading in JPYUSD for the direct trade and for the triangular trade. Please refer to Fig. 1.

Direct effect. Since there is more noise trading in JPYUSD market ($\sigma_{u_1} \nearrow$), the insider trader can better camouflage his information-driven trades in this market and will trade more aggressively, especially on the direct JPYUSD signal ($\beta_{11}^{+} \nearrow$). However, as the price impact in the JPYUSD market becomes smaller ($\lambda_1 \searrow$), the insider trader can also afford to trade more aggressively on the indirect USDEUR signal ($\beta_{12}^{-} \searrow$), through the triangle trade.

Triangular effect. Since the insider trades more over the triangle in the JPYUSD market ($\beta_{12}^{-} \searrow$), she will have to do it also in the JPYEUR market ($\beta_{32}^{+} \nearrow$). Because of limited risk-bearing capacity, as the insider's direct trades become more and more aggressive, trading through the triangle becomes less attractive because of mean-variance preferences[13].

Limited risk-bearing capacity also explains why price impact in USDEUR does not go up (λ_2 not $\nearrow$) as $\beta_{21}^{-} \to 0$ (despite comparatively less noise trading in this market). Trading a lot on the JPYUSD signal makes trading on the USDEUR signal relatively less attractive. Such "substitutability" of the signals from the perspective of risk should not hold in a model with two insider traders, each with a different signal (e.g., one trader

[13]Note that $\beta_{21}^{-} \to 0$ and $\beta_{31}^{+} \to 0$ of course. They load on the same risk source/signal as β_{11}^{+}. However, even $\beta_{22}^{+} \searrow$ a little, despite loading on a uncorrelated risk.

receives a signal on the JPYUSD rate, the other on the USDEUR rate)[14].

Lastly, the figure allows to compare the strategic triangular trading model with a benchmark model where only direct trading is allowed. In the benchmark model, the insider trader can express her views on the appreciation or depreciation of a currency pair only by trading in that same market for the pair. She cannot trade over the triangle. As expected for the benchmark model, β_2, β_3 and λ_2, λ_3 (dotted lines) are all constant (since σ_{u_1} does not show up in these markets).

B. Increased information imprecision on JPYUSD

Market liquidity, in the sense of market depth, is sensitive to the value of the private information. The value of the private information captures the utility of a private signal. From the perspective of the insider trader who receives a private signal, the information content becomes more valuable as a signal becomes more precise and asymmetric. The precision and asymmetry of the signal help the insider trader to consolidate her advantageous position as information monopolist. Since the original Kyle (1985)'s model already captures the effect of information asymmetry, it is more interesting to investigate the effect of information imprecision. The following discussion is focused on the effects of increased information imprecision on JPYUSD for the direct trade and for the triangular trade. Please refer to Fig. 2.

Direct effect. If the imprecision on the JPYUSD signal becomes extremely large (i.e., $\sigma_{\epsilon_1} > 15$), signal-to-noise ratio becomes so small that the insider trader has no more information on JPYUSD and will not trade at all on it ($\beta_{11}^{+} \to 0$). Thus, as the noise increases, the insider trades less and less aggressively, so the JPYUSD's market maker learns less from the order flow, and the reflected price adjustment will be smaller ($\lambda_1 \searrow$).

Triangular effect. As the JPYUSD signal becomes more and more noisy ($\sigma_{\epsilon_1} \nearrow$), the insider is not going to trade even over the triangle ($\beta_{31}^{+} \to 0$ and $\beta_{21}^{-} \to 0$ at the same rate). It is not a coincidence that the parameter β_{22}^{+} stabilizes exactly when the noise is

[14]In such differential information model, if one trader takes a lot of risk on his own signal, the other trader might do the same as well. See Paul (1993) for a discussion on crowding-out effects and the informativeness of security prices.

at the same level for which $\beta_{31}^{+} \to 0$ and $\beta_{21}^{-} \to 0$. The intuition is the following. When the signal is still precise enough (i.e., $\sigma_{\epsilon_1} < 4$), the insider still trades over the triangle based on the JPYUSD signal (β_{31}^{+} and β_{21}^{-} are not yet vanished). Then, the USDEUR's market maker still does not know, when observing the pooled order flow, if the flow includes the direct trade on the signal USDEUR (the only signal she cares about) or the triangle trade on the signal JPYUSD. Therefore, the market maker learns less from the order flow and the insider trader can trade more aggressively on the USDEUR signal (β_{22}^{+} is relatively high). Overall, the price impact in USDEUR (λ_2) is kept mostly stable by these two offsetting effect, i.e. by the confusion of the market maker.

However, as soon as the JPYUSD signal's imprecision is large enough (i.e., $\sigma_{\epsilon_1} > 6$), the market maker knows that the insider is certainly not trading on JPYUSD over the triangle. All the order flow that the market maker observes in the USDEUR market must be the direct and informed trade on the USDEUR signal (or just noise trading). Therefore, the insider cannot afford anymore to trade on the USDEUR signal as aggressively as when the market maker was still confused (β_{22}^{+} stabilizes at a lower level).

Moreover, since the price impact $\lambda_1 \searrow$ a lot and very quickly as $\sigma_{\epsilon_1} \nearrow$, the insider finds more and more attractive trading over the triangle and can trade much more aggressively (as the price wouldn't move against her a lot). This explains why $|\beta_{12}^{-}| \nearrow$ significantly. The following is the reason why the market maker in JPYUSD does not penalize the insider for trading this aggressively. The more $\sigma_{\epsilon_1} \nearrow$, the more certain the market maker is that the insider is just trading the other signal over the triangle and not on information the market maker cares about (i.e., JPYUSD signal). This reinforces the drop in price impact ($\lambda_1 \searrow$).

Since the insider is now trading over the triangle more aggressively, also $\beta_{32}^{+} \nearrow$. Moreover, if USD appreciates against JPY and EUR appreciates against USD, then EUR will appreciate against JPY (β_{31}^{+} and β_{32}^{+} are additive). Therefore, because $\beta_{31}^{+} \searrow$ quickly and $\beta_{32}^{+} \nearrow$ slowly, the price impact in JPYEUR (λ_3) will be hump-shaped.

Lastly, the figure allows to compare the strategic triangular trading model with the benchmark model where only direct trading is allowed. In the benchmark model, the insider trader can express her views on the appreciation or depreciation of a currency pair only by trading in that same market for the pair. She cannot trade over the triangle. As expected for the benchmark model, β_2 and λ_2 (dotted red lines) are of course insensitive to σ_{ϵ_1}, thus they remain constant.

C. Strategic interaction channels driving the behaviour of the agents

The findings discussed in the previous two subsections bring to light two underlying mechanisms. These are the two strategic channels which regulate the interactions between the insider trader and the market makers.

The first channel is the *limited risk-bearing capacity of the insider trader*. The insider trader has mean-variance preferences, with the variance being convex. This means that the insider is willing to take a certain amount of risk but not more than that. She can distribute such risk asymmetrically by trading either directly or indirectly. Then, the more aggressive is the direct trade, the less attractive the triangle trade becomes. Without mean-variance preferences the relative attractiveness would be unaffected.

The second channel is the *limited cross-learning capacity of the market maker*. Assuming zero correlation between two signals $\zeta_{\frac{¥}{\$}}$ and $\zeta_{\frac{\$}{€}}$, trading the signal $\zeta_{\frac{¥}{\$}}$ over the triangle is strategic for the insider because it makes it more difficult for the market maker who wants to learn about the other signal, i.e. signal $\zeta_{\frac{\$}{€}}$. From the perspective of such market maker, triangle trading on the signal $\zeta_{\frac{¥}{\$}}$ is like noise trading. However, as the signal $\zeta_{\frac{¥}{\$}}$ becomes less valuable, the triangle trade on $\zeta_{\frac{¥}{\$}}$ becomes less likely. In turn, the same market maker would expect direct trading on $\zeta_{\frac{\$}{€}}$ to become more likely, driving up price impact and dampening market liquidity in the market for the USDEUR rate.

Overall, the two strategic interaction channels provide a rich characterization of market liquidity and informed trading in a triangular trading setting where currency speculators are allowed to implement both direct and triangle trades. This is a novel theoretical finding, with the potential to contribute in explaining the actual behaviour of better informed FX traders in real financial markets.

IV. Conclusion

In the FX market large liquidity coexists with systemic asymmetric information. This fact is counter-intuitive from the perspective of the asymmetric information theory.

The paper characterizes the strategic interaction between an insider trader and market makers in a triangular FX trading setting. In the strategic model, the insider can trade currency pairs both directly and indirectly, through a third currency. In each of the three markets, the insider trader takes into account also the market liquidity in the other two markets when choosing the optimal trading strategy.

The cross-market sensitivity of the insider's trading aggressiveness and the market makers' price adjustments to the noise trading and to the information features reveals (i) limited risk-bearing capacity of the insider trader, and (ii) limited cross-learning capacity of the market makers. These two underlying strategic interaction channels could contribute in explaining the actual behaviour of better informed high-frequency trading firms and hedge funds, who have become major determinants of cross-market FX liquidity.

Moreover, a solid theoretical understanding of FX market liquidity could help policy makers to better address financial stability risks affecting entire currency networks. For example, currency crises are associated with sudden and dramatic drains in liquidity, and some tend to propagate across markets through illiquidity spillovers. Despite currency crises emerge from the microstructure of financial markets, they have macroeconomic relevance and implications. In future research, the strategic triangular trading model could be tailored to explicitly investigate liquidity commonality and spillovers between different FX spot markets.

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Figures

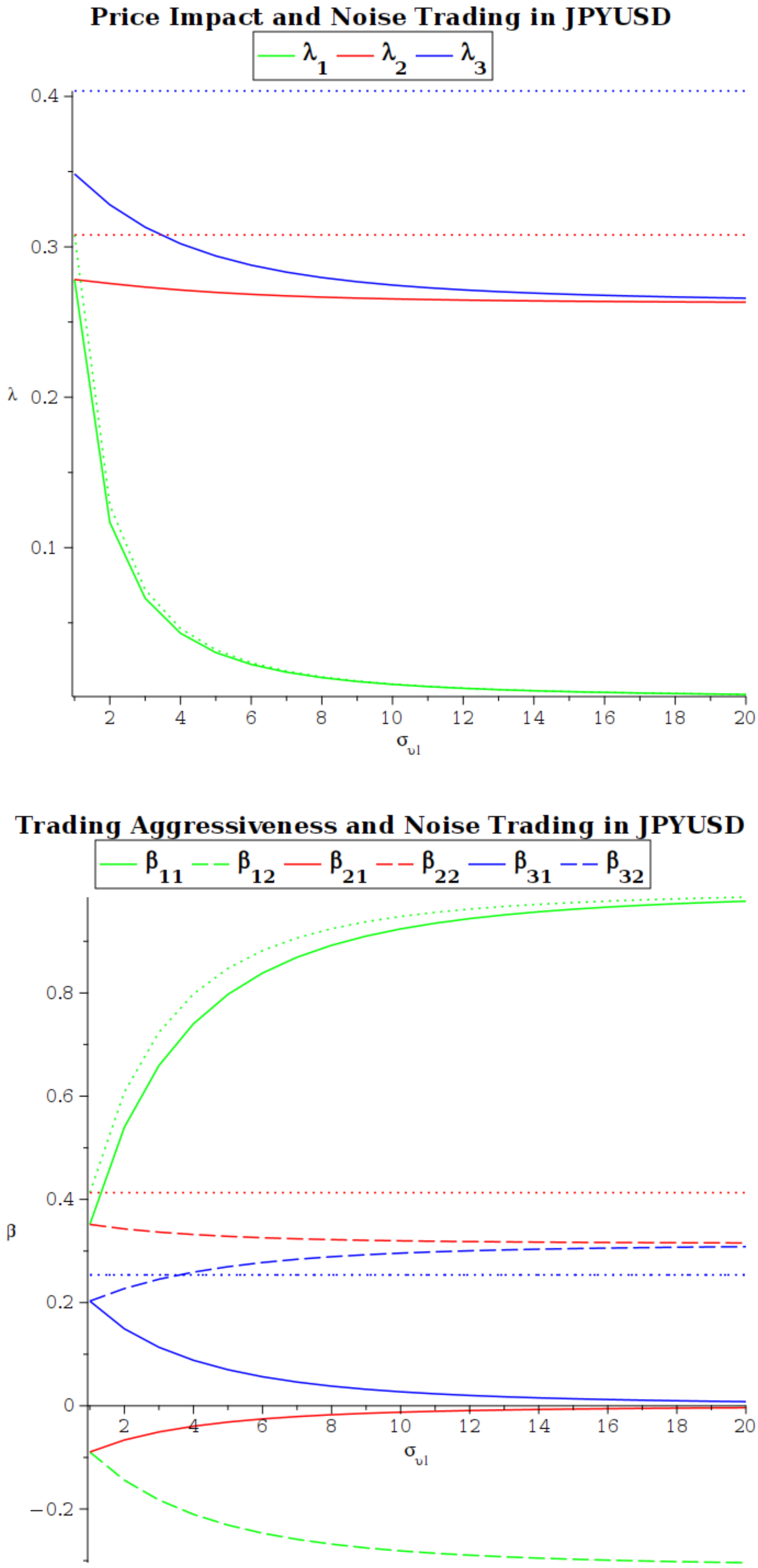

Figure 1: Price impact (top) and trading aggressiveness (bottom) with increasing noise trading in JPYUSD (σ_{u_1}). Reference equations: $\chi_{\frac{\text{¥}}{\$}} \sim \beta^{+}_{11}\zeta_{\frac{\text{¥}}{\$}} + \beta^{-}_{12}\zeta_{\frac{\$}{\text{€}}}$, $\chi_{\frac{\$}{\text{€}}} \sim \beta^{-}_{21}\zeta_{\frac{\text{¥}}{\$}} + \beta^{+}_{22}\zeta_{\frac{\$}{\text{€}}}$, $\chi_{\frac{\text{¥}}{\text{€}}} \sim \beta^{+}_{31}\zeta_{\frac{\text{¥}}{\$}} + \beta^{+}_{32}\zeta_{\frac{\$}{\text{€}}}$.

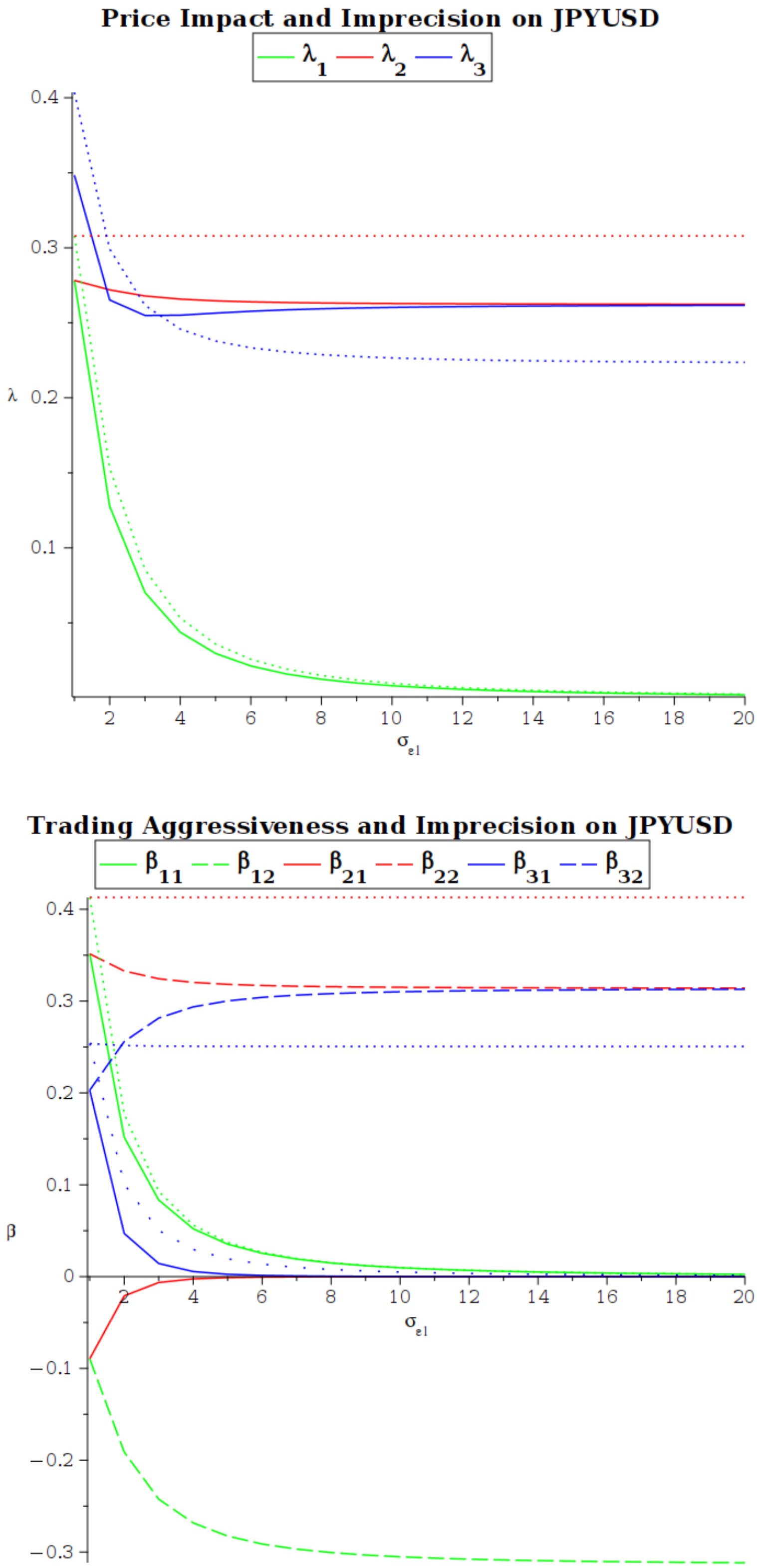

Figure 2: Price impact (top) and trading aggressiveness (bottom) with increasing information imprecision on JPYUSD (σ_{ϵ_1}). Reference equations: $\chi_{\frac{¥}{\$}} \sim \beta^{+}_{11}\zeta_{\frac{¥}{\$}} + \beta^{-}_{12}\zeta_{\frac{\$}{€}}$, $\chi_{\frac{\$}{€}} \sim \beta^{-}_{21}\zeta_{\frac{¥}{\$}} + \beta^{+}_{22}\zeta_{\frac{\$}{€}}$, $\chi_{\frac{¥}{€}} \sim \beta^{+}_{31}\zeta_{\frac{¥}{\$}} + \beta^{+}_{32}\zeta_{\frac{\$}{€}}$.

V. Appendix

A. Discussion on the market liquidity of currency markets

Market liquidity is bi-faceted, as it accounts for two different properties of markets: tightness and depth. In the words of Kyle (1985), market tightness is "the cost of turning around a position over a short period of time", while market depth is "the size of an order flow innovation required to change prices a given amount".

Market tightness was the focus of the earlier literature on FX market liquidity (e.g., Huang and Masulis (1999) and Karnaukh, Ranaldo, and Soderlind (2015)). Market tightness is straightforward to quantify: it is measured by transaction costs, and these are directly observable in the form of FX bid-ask spreads. However, the order-driven triangular trading model introduced by this paper does not allow for bid-ask spreads because the placement of orders happens before the quotes are set[15].

Market depth can be intuitively associated to Kyle (1985)'s lambda, but the theoretical concept hardly translates into a testable measure. Past attempts have led to mixed results. See, for example, the market microstructure invariance hypotheses introduced in Kyle and Obizhaeva (2016) or the highly contested VPIN measure of Easley, Lopez de Prado, and O'Hara (2012).

Moreover, the lack of comprehensive data on FX volume or order flow hindered not only the development of FX-specific proxies for market depth, but even the adaptations of approximate measures popularly used for stocks, such as Amihud (2002)'s[16]. Thanks to the advent of CLS Group's datasets, this is no longer the case. Hasbrouck and Levich (2019) are pioneers in their proposal of an Amihud measure for FX trading.

Ranaldo and Santucci de Magistris (2022) refine the measure in order to study global FX liquidity in relation to violations of the triangular no-arbitrage condition. The authors introduce the "realized Amihud" estimator, which quantifies the FX volatility at-

[15]Only quote-driven trading models would allow to measure market tightness using FX bid-ask spreads. In quote-driven trading models, market makers set bid and ask prices and then traders submit orders. For stocks, the canonical example is the theoretical model by Glosten and Milgrom (1985).

[16]Chordia, Huh, and Subrahmanyam (2009) and Foucault, Pagano, and Roell (2013) show that Amihud (2002)'s stock illiquidity measure approximates fairly well Kyle (1985)'s lambda.

tributed to each unit of trading volume[17]. In their model, market depth is a positive slope constant that captures the market's capability to accommodate substantial trading volumes at the convergence of demand and supply. The steeper the slope of the demand curve for a given supply, the larger the price impact of a given volume size and the higher the "realized Amihud".

The novel FX volume-based Amihud measures are specifically designed for currencies and can now provide a reliable testing ground for the triangular trading model introduced in this paper. However, these measures could potentially capture the endogenous market liquidity derived in the theoretical model (i.e., the reciprocals of (3)) only if the orders take a well-defined direction. In fact, trading volume is highly correlated to order flow imbalance only when orders are predominantly buy or sell orders.

[17] They also hint at enhancing the robustness of the estimator by using Darolles, Le Fol, and Mero (2017)'s dynamic extension of the mixture of distribution hypothesis.

B. Normal distributions of the random variables

Consider, for example, the exchange rates JPYUSD and USDEUR. The random variable for the private signal $\widetilde{\zeta} = \widetilde{\nu} + \widetilde{\epsilon}$ about the two log-exchange rates $\widetilde{\nu}$ ($\perp \widetilde{\epsilon}$) is normally distributed:

$$\begin{bmatrix} \widetilde{\zeta}_{\frac{¥}{\$}} \\ \widetilde{\zeta}_{\frac{\$}{€}} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \overline{\nu}_{\frac{¥}{\$}} \\ \overline{\nu}_{\frac{\$}{€}} \end{bmatrix}, \begin{bmatrix} \sigma^2_{\nu_{\frac{¥}{\$}}} + \sigma^2_{\epsilon_{\frac{¥}{\$}}} & \sigma_{\nu_{\frac{¥}{\$}}} \rho_{\nu_{\frac{¥}{\$},\frac{\$}{€}}} \sigma_{\nu_{\frac{\$}{€}}} \\ \sigma_{\nu_{\frac{¥}{\$}}} \rho_{\nu_{\frac{¥}{\$},\frac{\$}{€}}} \sigma_{\nu_{\frac{\$}{€}}} & \sigma^2_{\nu_{\frac{\$}{€}}} + \sigma^2_{\epsilon_{\frac{\$}{€}}} \end{bmatrix} \right)$$

$$\begin{bmatrix} \widetilde{\nu}_{\frac{¥}{\$}} \\ \widetilde{\nu}_{\frac{\$}{€}} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \overline{\nu}_{\frac{¥}{\$}} \\ \overline{\nu}_{\frac{\$}{€}} \end{bmatrix}, \begin{bmatrix} \sigma^2_{\nu_{\frac{¥}{\$}}} & \sigma_{\nu_{\frac{¥}{\$}}} \rho_{\nu_{\frac{¥}{\$},\frac{\$}{€}}} \sigma_{\nu_{\frac{\$}{€}}} \\ \sigma_{\nu_{\frac{¥}{\$}}} \rho_{\nu_{\frac{¥}{\$},\frac{\$}{€}}} \sigma_{\nu_{\frac{\$}{€}}} & \sigma^2_{\nu_{\frac{\$}{€}}} \end{bmatrix} \right) \quad \begin{bmatrix} \widetilde{\epsilon}_{\frac{¥}{\$}} \\ \widetilde{\epsilon}_{\frac{\$}{€}} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{\epsilon_{\frac{¥}{\$}}} \\ \sigma^2_{\epsilon_{\frac{\$}{€}}} \end{bmatrix} \right)$$

Also normally distributed is the price-inelastic demand $\widetilde{u}$ of the noise traders in the three markets for currency pairs JPYUSD, USDEUR and JPYEUR:

$$\begin{bmatrix} \widetilde{u}_{\frac{¥}{\$}} \\ \widetilde{u}_{\frac{\$}{€}} \\ \widetilde{u}_{\frac{¥}{€}} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{u_{\frac{¥}{\$}}} & \sigma_{u_{\frac{¥}{\$}}} \rho_{u_{\frac{¥}{\$},\frac{\$}{€}}} \sigma_{u_{\frac{\$}{€}}} & \sigma_{u_{\frac{¥}{\$}}} \rho_{u_{\frac{¥}{\$},\frac{¥}{€}}} \sigma_{u_{\frac{¥}{€}}} \\ \sigma_{u_{\frac{¥}{\$}}} \rho_{u_{\frac{¥}{\$},\frac{\$}{€}}} \sigma_{u_{\frac{\$}{€}}} & \sigma^2_{u_{\frac{\$}{€}}} & \sigma_{u_{\frac{\$}{€}}} \rho_{u_{\frac{\$}{€},\frac{¥}{€}}} \sigma_{u_{\frac{¥}{€}}} \\ \sigma_{u_{\frac{¥}{\$}}} \rho_{u_{\frac{¥}{\$},\frac{¥}{€}}} \sigma_{u_{\frac{¥}{€}}} & \sigma_{u_{\frac{\$}{€}}} \rho_{u_{\frac{\$}{€},\frac{¥}{€}}} \sigma_{u_{\frac{¥}{€}}} & \sigma^2_{u_{\frac{¥}{€}}} \end{bmatrix} \right)$$

C. Approximated combined profits in USD

Recall that $\chi_{\frac{\text{¥}}{\$}}$ is the USD trade quantity, $\chi_{\frac{\$}{\text{€}}}$ is the USD volume invested in EUR, and $\chi_{\frac{\text{¥}}{\text{€}}}$ is the USD value of the EUR trade in the JPYEUR market. Denote as q the current exchange rates and as f (just for now) the future exchange rates. Then, the profits denominated in USD are:

$$
\begin{aligned}
\pi_{\frac{\text{¥}}{\$}} &\overset{\text{¥}}{=} \chi_{\frac{\text{¥}}{\$}}(f_{\frac{\text{¥}}{\$}} - q_{\frac{\text{¥}}{\$}}) \overset{\$}{=} \chi_{\frac{\text{¥}}{\$}}(1 - q_{\frac{\text{¥}}{\$}}/f_{\frac{\text{¥}}{\$}}) \\
\pi_{\frac{\$}{\text{€}}} &\overset{\$}{=} (\chi_{\frac{\$}{\text{€}}}/q_{\frac{\$}{\text{€}}})(f_{\frac{\$}{\text{€}}} - q_{\frac{\$}{\text{€}}}) = \chi_{\frac{\$}{\text{€}}}(f_{\frac{\$}{\text{€}}}/q_{\frac{\$}{\text{€}}} - 1) \\
\pi_{\frac{\text{¥}}{\text{€}}} &\overset{\text{¥}}{=} (\chi_{\frac{\text{¥}}{\text{€}}}/q_{\frac{\$}{\text{€}}})(f_{\frac{\text{¥}}{\text{€}}} - q_{\frac{\text{¥}}{\text{€}}}) \overset{\$}{=} (\chi_{\frac{\text{¥}}{\text{€}}}/q_{\frac{\$}{\text{€}}})(f_{\frac{\text{¥}}{\text{€}}} - q_{\frac{\text{¥}}{\text{€}}})/f_{\frac{\text{¥}}{\$}} \\
&\overset{\text{no arb}}{=} \chi_{\frac{\text{¥}}{\text{€}}}(f_{\frac{\text{¥}}{\$}} f_{\frac{\$}{\text{€}}} - q_{\frac{\text{¥}}{\text{€}}})/(f_{\frac{\text{¥}}{\$}} q_{\frac{\$}{\text{€}}}) = \chi_{\frac{\text{¥}}{\text{€}}}(f_{\frac{\$}{\text{€}}}/q_{\frac{\$}{\text{€}}} - 1 - (q_{\frac{\text{¥}}{\text{€}}}/f_{\frac{\text{¥}}{\$}} q_{\frac{\$}{\text{€}}} - 1))
\end{aligned}
$$

After log-linearization ($f/q - 1 \approx \ln(f/q)$ for $f \approx q$), the approximated profits denominated in USD become:

$$
\begin{aligned}
\pi^*_{\frac{\text{¥}}{\$}} &= \chi_{\frac{\text{¥}}{\$}} \ln(f_{\frac{\text{¥}}{\$}}/q_{\frac{\text{¥}}{\$}}) = \chi_{\frac{\text{¥}}{\$}}(\ln f_{\frac{\text{¥}}{\$}} - \ln q_{\frac{\text{¥}}{\$}}) \\
\pi^*_{\frac{\$}{\text{€}}} &= \chi_{\frac{\$}{\text{€}}} \ln(f_{\frac{\$}{\text{€}}}/q_{\frac{\$}{\text{€}}}) = \chi_{\frac{\$}{\text{€}}}(\ln f_{\frac{\$}{\text{€}}} - \ln q_{\frac{\$}{\text{€}}}) \\
\pi^*_{\frac{\text{¥}}{\text{€}}} &= \chi_{\frac{\text{¥}}{\text{€}}}(\ln(f_{\frac{\$}{\text{€}}}/q_{\frac{\$}{\text{€}}}) - \ln(q_{\frac{\text{¥}}{\text{€}}}/f_{\frac{\text{¥}}{\$}} q_{\frac{\$}{\text{€}}})) = \chi_{\frac{\text{¥}}{\text{€}}}(\ln f_{\frac{\text{¥}}{\$}} + \ln f_{\frac{\$}{\text{€}}} - \ln q_{\frac{\text{¥}}{\text{€}}})
\end{aligned}
$$

The insider is strategic and incorporates the conjectured quoting rules:

$$
\begin{aligned}
\pi^*_{\frac{\text{¥}}{\$}} &= \chi_{\frac{\text{¥}}{\$}}(v_{\frac{\text{¥}}{\$}} - (\overline{v}_{\frac{\text{¥}}{\$}} + \lambda_{\frac{\text{¥}}{\$}}(u_{\frac{\text{¥}}{\$}} + \chi_{\frac{\text{¥}}{\$}}))) \\
\pi^*_{\frac{\$}{\text{€}}} &= \chi_{\frac{\$}{\text{€}}}(v_{\frac{\$}{\text{€}}} - (\overline{v}_{\frac{\$}{\text{€}}} + \lambda_{\frac{\$}{\text{€}}}(u_{\frac{\$}{\text{€}}} + \chi_{\frac{\$}{\text{€}}}))) \\
\pi^*_{\frac{\text{¥}}{\text{€}}} &= \chi_{\frac{\text{¥}}{\text{€}}}(v_{\frac{\text{¥}}{\$}} + v_{\frac{\$}{\text{€}}} - (\overline{v}_{\frac{\text{¥}}{\$}} + \overline{v}_{\frac{\$}{\text{€}}} + \lambda_{\frac{\text{¥}}{\text{€}}}(u_{\frac{\text{¥}}{\text{€}}} + \chi_{\frac{\text{¥}}{\text{€}}})))
\end{aligned}
$$

Overall, the approximated combined profits denominated in USD can be compactly re-written as a linear combination of the log-exchange rates $v_{\frac{\text{¥}}{\$}}$ and $v_{\frac{\$}{\text{€}}}$, and of the noise trading quantities $u_{\frac{\text{¥}}{\$}}$, $u_{\frac{\$}{\text{€}}}$, and $u_{\frac{\text{¥}}{\text{€}}}$:

$$
\Pi^* = \pi^*_{\frac{\text{¥}}{\$}} + \pi^*_{\frac{\$}{\text{€}}} + \pi^*_{\frac{\text{¥}}{\text{€}}} = a + b v_{\frac{\text{¥}}{\$}} + c v_{\frac{\$}{\text{€}}} + d u_{\frac{\text{¥}}{\$}} + e u_{\frac{\$}{\text{€}}} + f u_{\frac{\text{¥}}{\text{€}}}
$$

with the coefficients as follows[18]:

$$a = \chi_{\frac{\yen}{\$}}\left(-\lambda_{\frac{\yen}{\$}}\chi_{\frac{\yen}{\$}} - \overline{\nu}_{\frac{\yen}{\$}}\right) + \chi_{\frac{\$}{\euro}}\left(-\lambda_{\frac{\$}{\euro}}\chi_{\frac{\$}{\euro}} - \overline{\nu}_{\frac{\$}{\euro}}\right) + \chi_{\frac{\yen}{\euro}}\left(-\lambda_{\frac{\yen}{\euro}}\chi_{\frac{\yen}{\euro}} - \overline{\nu}_{\frac{\yen}{\$}} - \overline{\nu}_{\frac{\$}{\euro}}\right)$$

$$b = \chi_{\frac{\yen}{\$}} + \chi_{\frac{\yen}{\euro}} \quad c = \chi_{\frac{\$}{\euro}} + \chi_{\frac{\yen}{\euro}} \quad d = -\lambda_{\frac{\yen}{\$}}\chi_{\frac{\yen}{\$}} \quad e = -\lambda_{\frac{\$}{\euro}}\chi_{\frac{\$}{\euro}} \quad f = -\lambda_{\frac{\yen}{\euro}}\chi_{\frac{\yen}{\euro}}$$

[18]From now on, f indicates the coefficient in the linear combination, and not a future exchange rate.

D. Insider trader problem

The insider trader mean-variance optimizes the approximated profits:

$$\max_{\chi_{\frac{\text{¥}}{\$}},\chi_{\frac{\$}{\text{€}}},\chi_{\frac{\text{¥}}{\text{€}}}}\left\{\mathrm{E}\left[\Pi^*|\zeta\right]-\frac{\gamma}{2}\mathrm{Var}\left[\Pi^*|\zeta\right]\right\}$$

$$\mathrm{E}\left[\Pi^*|\zeta\right]=a+b\mathrm{E}\left[\nu_{\frac{\text{¥}}{\$}}|\zeta\right]+c\mathrm{E}\left[\nu_{\frac{\$}{\text{€}}}|\zeta\right]$$

$$\begin{aligned}\mathrm{Var}\left[\Pi^*|\zeta\right]&=b^2\mathrm{Var}\left[\nu_{\frac{\text{¥}}{\$}}|\zeta\right]+c^2\mathrm{Var}\left[\nu_{\frac{\$}{\text{€}}}|\zeta\right]+2bc\mathrm{Cov}\left[\nu_{\frac{\text{¥}}{\$}},\nu_{\frac{\$}{\text{€}}}|\zeta\right]\\&\quad+d^2\mathrm{Var}\left[u_{\frac{\text{¥}}{\$}}\right]+e^2\mathrm{Var}\left[u_{\frac{\$}{\text{€}}}\right]+f^2\mathrm{Var}\left[u_{\frac{\text{¥}}{\text{€}}}\right]\\&\quad+2de\mathrm{Cov}\left[u_{\frac{\text{¥}}{\$}},u_{\frac{\$}{\text{€}}}\right]+2df\mathrm{Cov}\left[u_{\frac{\text{¥}}{\$}},u_{\frac{\text{¥}}{\text{€}}}\right]+2ef\mathrm{Cov}\left[u_{\frac{\$}{\text{€}}},u_{\frac{\text{¥}}{\text{€}}}\right]\end{aligned}$$

The conditional mean and variance are found by applying the linear projection theorem:

$$\mathrm{E}\left[\nu|\zeta\right]=\mu_\nu+\Sigma_{\nu\zeta}\Sigma_\zeta^{-1}(\zeta-\mu_\zeta)=\mu_\nu+\Sigma_\nu\Sigma_\zeta^{-1}(\zeta-\mu_\zeta)=\begin{bmatrix}\mathrm{E}\left[\nu_{\frac{\text{¥}}{\$}}|\zeta\right]\\\mathrm{E}\left[\nu_{\frac{\$}{\text{€}}}|\zeta\right]\end{bmatrix}$$

$$\mathrm{Var}\left[\nu|\zeta\right]=\Sigma_\nu-\Sigma_{\nu\zeta}\Sigma_\zeta^{-1}\Sigma_{\zeta\nu}=\Sigma_\nu-\Sigma_\nu\Sigma_\zeta^{-1}\Sigma_\nu=\begin{bmatrix}\mathrm{Var}\left[\nu_{\frac{\text{¥}}{\$}}|\zeta\right]&\mathrm{Cov}\left[\nu_{\frac{\text{¥}}{\$}},\nu_{\frac{\$}{\text{€}}}|\zeta\right]\\\mathrm{Cov}\left[\nu_{\frac{\text{¥}}{\$}},\nu_{\frac{\$}{\text{€}}}|\zeta\right]&\mathrm{Var}\left[\nu_{\frac{\$}{\text{€}}}|\zeta\right]\end{bmatrix}$$

Note that the conditional mean is linear in the signal deviations:

$$\mathrm{E}\left[\nu_{\frac{\text{¥}}{\$}}|\zeta\right]=\overline{\nu}_{\frac{\text{¥}}{\$}}+\alpha_{11}(\zeta_{\frac{\text{¥}}{\$}}-\overline{\nu}_{\frac{\text{¥}}{\$}})+\alpha_{12}(\zeta_{\frac{\$}{\text{€}}}-\overline{\nu}_{\frac{\$}{\text{€}}})$$

$$\mathrm{E}\left[\nu_{\frac{\$}{\text{€}}}|\zeta\right]=\overline{\nu}_{\frac{\$}{\text{€}}}+\alpha_{21}(\zeta_{\frac{\text{¥}}{\$}}-\overline{\nu}_{\frac{\text{¥}}{\$}})+\alpha_{22}(\zeta_{\frac{\$}{\text{€}}}-\overline{\nu}_{\frac{\$}{\text{€}}})$$

The coefficients in the conditional mean and variance are easily found.

Define the common factor:

$$\phi^{-1}=\mathrm{Var}[\zeta_{\frac{\text{¥}}{\$}}]\mathrm{Var}[\zeta_{\frac{\$}{\text{€}}}]-\mathrm{Cov}^2[\zeta_{\frac{\text{¥}}{\$}},\zeta_{\frac{\$}{\text{€}}}]$$

Then, just one of these coefficients, e.g. $\alpha_{12} = \phi \mathrm{Var}[\epsilon_{\frac{¥}{\$}}]\mathrm{Cov}[\zeta_{\frac{¥}{\$}}, \zeta_{\frac{\$}{€}}]$, pins down all the others. The coefficients in the mean are:

$$\alpha_{11} = 1 - \alpha_{12}\frac{\mathrm{Var}[\zeta_{\frac{\$}{€}}]}{\mathrm{Cov}[\zeta_{\frac{¥}{\$}}, \zeta_{\frac{\$}{€}}]} \quad \alpha_{21} = \alpha_{12}\frac{\mathrm{Var}[\epsilon_{\frac{\$}{€}}]}{\mathrm{Var}[\epsilon_{\frac{¥}{\$}}]} \quad \alpha_{22} = 1 - \alpha_{21}\frac{\mathrm{Var}[\zeta_{\frac{¥}{\$}}]}{\mathrm{Cov}[\zeta_{\frac{¥}{\$}}, \zeta_{\frac{\$}{€}}]}$$

The coefficients in the variance are:

$$\mathrm{Var}[\nu_{\frac{¥}{\$}}|\zeta] = \alpha_{12}\frac{\mathrm{Var}[\nu_{\frac{¥}{\$}}]\mathrm{Var}[\zeta_{\frac{\$}{€}}] - \mathrm{Cov}^2[\zeta_{\frac{¥}{\$}}, \zeta_{\frac{\$}{€}}]}{\mathrm{Cov}[\zeta_{\frac{¥}{\$}}, \zeta_{\frac{\$}{€}}]} \quad \mathrm{Cov}[\nu_{\frac{¥}{\$}}, \nu_{\frac{\$}{€}}|\zeta] = \alpha_{12}\mathrm{Var}[\epsilon_{\frac{\$}{€}}]$$

$$\mathrm{Var}[\nu_{\frac{\$}{€}}|\zeta] = \alpha_{21}\frac{\mathrm{Var}[\nu_{\frac{\$}{€}}]\mathrm{Var}[\zeta_{\frac{¥}{\$}}] - \mathrm{Cov}^2[\zeta_{\frac{¥}{\$}}, \zeta_{\frac{\$}{€}}]}{\mathrm{Cov}[\zeta_{\frac{¥}{\$}}, \zeta_{\frac{\$}{€}}]}$$

E. Market maker problem

The simplifying assumption is that market makers are separate across the three currency markets. The predicted quoting rules, given perfect competition in each market, follow from the linear projection theorem:

$$\ln q^*_{\frac{¥}{\$}} = \mathrm{E}\left[\nu_{\frac{¥}{\$}} | \chi_{\frac{¥}{\$}} + u_{\frac{¥}{\$}}\right] = \mathrm{E}\left[\nu_{\frac{¥}{\$}}\right] + \frac{\mathrm{Cov}\left[\nu_{\frac{¥}{\$}}, \chi_{\frac{¥}{\$}} + u_{\frac{¥}{\$}}\right]}{\mathrm{Var}\left[\chi_{\frac{¥}{\$}} + u_{\frac{¥}{\$}}\right]}\left(\chi_{\frac{¥}{\$}} + u_{\frac{¥}{\$}} - \mathrm{E}\left[\chi_{\frac{¥}{\$}} + u_{\frac{¥}{\$}}\right]\right)$$

$$\ln q^*_{\frac{\$}{€}} = \mathrm{E}\left[\nu_{\frac{\$}{€}} | \chi_{\frac{\$}{€}} + u_{\frac{\$}{€}}\right] = \mathrm{E}\left[\nu_{\frac{\$}{€}}\right] + \frac{\mathrm{Cov}\left[\nu_{\frac{\$}{€}}, \chi_{\frac{\$}{€}} + u_{\frac{\$}{€}}\right]}{\mathrm{Var}\left[\chi_{\frac{\$}{€}} + u_{\frac{\$}{€}}\right]}\left(\chi_{\frac{\$}{€}} + u_{\frac{\$}{€}} - \mathrm{E}\left[\chi_{\frac{\$}{€}} + u_{\frac{\$}{€}}\right]\right)$$

$$\ln q^*_{\frac{¥}{€}} = \mathrm{E}\left[\nu_{\frac{¥}{\$}} + \nu_{\frac{\$}{€}} | \chi_{\frac{¥}{€}} + u_{\frac{¥}{€}}\right] = \mathrm{E}\left[\nu_{\frac{¥}{\$}} + \nu_{\frac{\$}{€}}\right] + \frac{\mathrm{Cov}\left[\nu_{\frac{¥}{\$}} + \nu_{\frac{\$}{€}}, \chi_{\frac{¥}{€}} + u_{\frac{¥}{€}}\right]}{\mathrm{Var}\left[\chi_{\frac{¥}{€}} + u_{\frac{¥}{€}}\right]}\left(\chi_{\frac{¥}{€}} + u_{\frac{¥}{€}} - \mathrm{E}\left[\chi_{\frac{¥}{€}} + u_{\frac{¥}{€}}\right]\right)$$

Recall that $\tilde{u} \perp \tilde{\nu}$, $\tilde{u} \perp \tilde{\chi}$, and $\mathrm{E}\left[\tilde{\chi}_i + \tilde{u}_i\right] = 0$, $\forall i \in \{\frac{¥}{\$}, \frac{\$}{€}, \frac{¥}{€}\}$. Price impacts are found by matching the predictions with the conjectured quoting rules (1):

$$\lambda_{\frac{¥}{\$}} = \frac{\mathrm{Cov}\left[\nu_{\frac{¥}{\$}}, \chi_{\frac{¥}{\$}}\right]}{\mathrm{Var}\left[\chi_{\frac{¥}{\$}}\right] + \mathrm{Var}\left[u_{\frac{¥}{\$}}\right]} \quad \lambda_{\frac{\$}{€}} = \frac{\mathrm{Cov}\left[\nu_{\frac{\$}{€}}, \chi_{\frac{\$}{€}}\right]}{\mathrm{Var}\left[\chi_{\frac{\$}{€}}\right] + \mathrm{Var}\left[u_{\frac{\$}{€}}\right]} \quad \lambda_{\frac{¥}{€}} = \frac{\mathrm{Cov}\left[\nu_{\frac{¥}{\$}}, \chi_{\frac{¥}{€}}\right] + \mathrm{Cov}\left[\nu_{\frac{\$}{€}}, \chi_{\frac{¥}{€}}\right]}{\mathrm{Var}\left[\chi_{\frac{¥}{€}}\right] + \mathrm{Var}\left[u_{\frac{¥}{€}}\right]}$$

Finally, the price impact equations (3) are derived by expanding the computations (recall that $\Sigma_{\nu\zeta} = \Sigma_{\zeta\nu} = \Sigma_\nu$):

$$\lambda_{\frac{¥}{\$}} = \frac{\beta_{11}\mathrm{Var}\left[\nu_{\frac{¥}{\$}}\right] + \beta_{12}\mathrm{Cov}\left[\nu_{\frac{¥}{\$}}, \nu_{\frac{\$}{€}}\right]}{\beta_{11}^2\mathrm{Var}\left[\zeta_{\frac{¥}{\$}}\right] + \beta_{12}^2\mathrm{Var}\left[\zeta_{\frac{\$}{€}}\right] + 2\beta_{11}\beta_{12}\mathrm{Cov}\left[\zeta_{\frac{¥}{\$}}, \zeta_{\frac{\$}{€}}\right] + \mathrm{Var}\left[u_{\frac{¥}{\$}}\right]}$$

$$\lambda_{\frac{\$}{€}} = \frac{\beta_{21}\mathrm{Cov}\left[\nu_{\frac{¥}{\$}}, \nu_{\frac{\$}{€}}\right] + \beta_{22}\mathrm{Var}\left[\nu_{\frac{\$}{€}}\right]}{\beta_{21}^2\mathrm{Var}\left[\zeta_{\frac{¥}{\$}}\right] + \beta_{22}^2\mathrm{Var}\left[\zeta_{\frac{\$}{€}}\right] + 2\beta_{21}\beta_{22}\mathrm{Cov}\left[\zeta_{\frac{¥}{\$}}, \zeta_{\frac{\$}{€}}\right] + \mathrm{Var}\left[u_{\frac{\$}{€}}\right]}$$

$$\lambda_{\frac{¥}{€}} = \frac{\beta_{31}\mathrm{Var}\left[\nu_{\frac{¥}{\$}}\right] + (\beta_{31} + \beta_{32})\mathrm{Cov}\left[\nu_{\frac{¥}{\$}}, \nu_{\frac{\$}{€}}\right] + \beta_{32}\mathrm{Var}\left[\nu_{\frac{\$}{€}}\right]}{\beta_{31}^2\mathrm{Var}\left[\zeta_{\frac{¥}{\$}}\right] + \beta_{32}^2\mathrm{Var}\left[\zeta_{\frac{\$}{€}}\right] + 2\beta_{31}\beta_{32}\mathrm{Cov}\left[\zeta_{\frac{¥}{\$}}, \zeta_{\frac{\$}{€}}\right] + \mathrm{Var}\left[u_{\frac{¥}{€}}\right]}$$